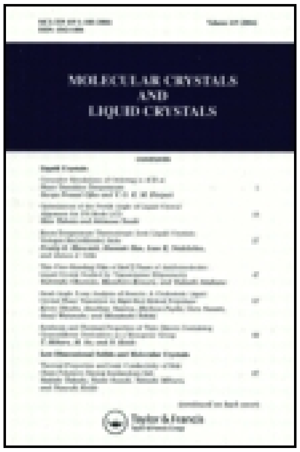

Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl19

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Version of record first published: 04 Oct 2006.

Mol. Cryst. Liq. Cryst. 1992, Vol. 223, pp. 241–249
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Printed in the United States of America

WAVE PROPAGATION IN LIQUID CRYSTAL SLAB WAVEGUIDES

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(Received November 6, 1991)

Abstract The propagation of guided electromagnetic waves in liquid crystal slab waveguides is considered. The dielectric properties of the liquid crystal layer which govern wave propagation are determined by the configuration of the nematic director. We examine TE and TM modes of a slab dielectric waveguide consisting of a liquid crystal layer which is both anisotropic and inhomogeneous. The uncoupled wave equations for these modes are derived; the equation describing the TM modes is complex. We show that a real propagation constant β exists for all frequencies above cutoff, and obtain the complex field distribution by solving the wave equation using an effective novel numerical technique.

INTRODUCTION

Liquid crystals have potential uses for optical waveguides[1,2] because of their unique response to optical fields. Optical fields can alter the dielectric properties of liquid crystals, and can therefore change the nature of the propagating modes. This behavior of liquid crystals makes them attractive candidates for nonlinear waveguide applications. As a first step, in this paper we consider linear waveguiding properties of liquid crystal slab waveguides.

Wave propagation in layered anisotropic media has been considered by a number of workers.[3,4] Since each layer is homogeneous, analytical solutions are available.[5] Here we consider slab waveguides consisting of liquid crystals which are both anisotropic and inhomogeneous. Such a system is shown schematically in Fig. 1. We first determine the configuration of the liquid crystal layer, and the dielectric tensor which depends on the director configuration. Next, the uncoupled wave equations are derived for the transverse electric (TE) and magnetic (TM) modes. We find that the coefficients in the equation describing the TM modes are complex. We then show that propagating modes exists for all frequencies; that is, the propagation constant β is real. For the TM mode, a modified shooting method is used to obtain numerical values of the field amplitude and phase as a function of position.

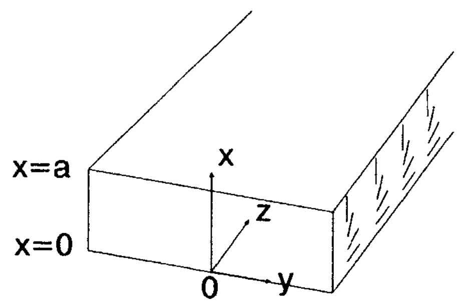

Figure 1. Schematic of the slab waveguide.

OPTICAL PROPERTIES OF LIQUID CRYSTALS

Dielectric properties of nematic liquid crystals depend on orientational order, and are determined by the director configuration. We consider a case where the director field $\hat{n}$ is given by

$$\hat{n} = (\sin\theta, 0, \cos\theta) \tag{1}$$

where θ is the angle between the nematic director and z-axis. We assume that θ only depends on the x coordinate, and that $\theta(0) = 0$, $\theta(a) = \frac{\pi}{2}$, as in the case of a typical hybrid cell. The free energy density of the system is[6]

$$F = \frac{1}{2} (K_1 \cos^2\theta + K_3 \sin^2\theta) (\frac{\partial\theta}{\partial x})^2 \tag{2}$$

where K_1 and K_3 are elastic constants. Minimizing the free energy gives rise to the Euler-Lagrange equation, which determines the director configuration and determines the dielectric tensor. If the director is in the x-z plane, the dielectric tensor is

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_{xx} & 0 & \epsilon_{xz} \\ 0 & \epsilon_{yy} & 0 \\ \epsilon_{zx} & 0 & \epsilon_{zz} \end{bmatrix} \tag{3}$$

where $\epsilon_{xx}=\epsilon_{\perp}+\Delta\epsilon\sin^2\theta$, $\epsilon_{xz}=\epsilon_{zx}=\Delta\epsilon\sin\theta\cos\theta$, $\epsilon_{yy}=\epsilon_{\perp}$, $\epsilon_{zz}=\epsilon_{\perp}+\Delta\epsilon\cos^2\theta$, and $\Delta\epsilon=\epsilon_{\parallel}-\epsilon_{\perp}$. $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ refer to the dielectric constants parallel and perpendicular to the local symmetry axis, respectively. Since θ is a function of position, the elements of the dielectric tensor vary in space.

For simplicity, we assume that $K_1=K_3$, and in this case $\theta=\frac{\pi}{2}(\frac{x}{a})$. However, the arguments below are valid for the general case where θ is an arbitrary smooth function of x.

SLAB WAVEGUIDE

A symmetric dielectric slab waveguide consists of a nematic liquid crystal layer of thickness a between two infinite isotropic regions with dielectric permittivity $\varepsilon_c=\epsilon_0\epsilon_c$; ϵ_0 is the permittivity of free space. The magnetic permeability μ is assumed to be the same as that of free space μ_0; the magnetic susceptibility of liquid crystals is negligible compared to the large dielectric permittivity. The permittivity of the liquid crystal layer is a spatially varying tensor given by Eq. 3. For wave guiding to take place, the average dielectric constant in the slab must be greater than ϵ_c. The structure of liquid crystal slab waveguide is shown in Fig. 1; wave propagation is assumed to be in the z direction.

For the waveguide considered, uncoupled TE and TM guided modes can exist. From Maxwell's equations, we obtain the wave equations for TE modes in the liquid crystal layer:

$$\frac{\partial^2 E_y}{\partial x^2} + (\omega^2 a^2 \mu_0 \epsilon_0 \epsilon_\perp - \beta^2 a^2) E_y = 0 \qquad (4)$$

$$H_x = -\frac{\beta a}{\omega a \mu_0} E_y \qquad (5)$$

$$H_z = \frac{i}{\omega a \mu_0} \frac{\partial E_y}{\partial x} \qquad (6)$$

where we have assumed the fields to have a time and z-coordinate dependence given by $e^{i(\omega t - \beta z)}$, here ω is the optical frequency, β is the propagation factor and the position x is measured in units of a. These equations may be solved analytically at once; the fields in the liquid crystal layer have sinusoidal spatial dependence, and fall off exponentially outside. The TE modes thus behave as in a homogeneous isotropic medium. Next, we consider the TM modes.

The wave equation for the TM modes in the liquid crystal layer becomes

$$\frac{\partial^2 H_y}{\partial x^2} + \left(\frac{1}{\epsilon_{xx}} \frac{\partial \epsilon_{xx}}{\partial x} - \frac{1}{D} \frac{\partial D}{\partial x} - 2i\beta a \frac{\epsilon_{xz}}{\epsilon_{xx}} \right) \frac{\partial H_y}{\partial x} +$$

$$\left[\omega^2 a^2 \mu_0 \epsilon_0 \frac{D}{\epsilon_{xx}} - \beta^2 a^2 \frac{\epsilon_{zz}}{\epsilon_{xx}} - i\beta a \frac{1}{\epsilon_{xx}} \frac{\partial \epsilon_{xz}}{\partial x} + i\beta a \frac{\epsilon_{xz}}{\epsilon_{xx}} \frac{1}{D} \frac{\partial D}{\partial x} \right] H_y = 0 \qquad (7)$$

where $D = \epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}^2$, and

$$E_x = \frac{1}{i\omega a \epsilon_0 D} \left[i\beta a \epsilon_{zz} H_y - \epsilon_{xz} \frac{\partial H_y}{\partial x} \right] \qquad (8)$$

$$E_z = \frac{1}{i\omega a \epsilon_0 D} \left[\epsilon_{xx} \frac{\partial H_y}{\partial x} - i\beta a \epsilon_{xz} H_y \right] \qquad (9)$$

Boundary conditions require the continuity of the tangential components of **E** and **H**. At x=0 and 1, H_y and E_z are continuous.

The complex coefficients in Eqs. 7-9 arise from the anisotropy and inhomogeneity of the liquid crystal layer. Eq. 7 is a nonlinear eigenvalue equation, where β is the eigenvalue. (We note that it is possible to transform Eq. 7 into a self-adjoint form; we chose, however, to stay with the current representation in terms of the field H_y.) For unattenuated guided waves the propagation constant β must be real; however it is not obvious that the eigenvalues β of Eq. 7 are real for all frequencies. To show that β is real, we rewrite Eq. 7 as

$$H_y'' + [(\ln\frac{\epsilon_{xx}}{D})' - 2i\beta a \frac{\epsilon_{xz}}{\epsilon_{xx}}]\, H_y' + [\omega^2 a^2 \mu_0 \epsilon_0 \frac{D}{\epsilon_{xx}} - \beta^2 a^2 \frac{\epsilon_{zz}}{\epsilon_{xx}} + i\beta a \frac{\epsilon_{xz}}{\epsilon_{xx}} (\ln\frac{D}{\epsilon_{xz}})']\, H_y = 0 \tag{10}$$

here primes indicate differentiation with respect to x. By multiplying Eq. 10 by H_y^* and subtracting the complex conjugate, and by making use of the identity $H_y^* H_y'' - H_y H_y^{*\prime\prime} = (H_y^* H_y' - H_y H_y^{*\prime})'$, we obtain

$$a^2(\beta^2 - \beta^{*2})(\frac{\epsilon_{zz}}{D}) H_y H_y^* + ia(\beta - \beta^*)\frac{\epsilon_{xz}}{D} W = [\frac{\epsilon_{xx}}{D} W - ia(\beta + \beta^*)\frac{\epsilon_{xz}}{D} H_y H_y^*]' \tag{11}$$

where $W \equiv H_y^* H_y' - H_y H_y^{*\prime}$. Integrating both sides of Eq. 11 gives

$$(\beta - \beta^*)[a^2(\beta + \beta^*)\int_0^1 \frac{\epsilon_{zz}}{D} H_y H_y^* \, dx + ia\int_0^1 \frac{\epsilon_{xz}}{D} W dx]$$

$$= [\frac{\epsilon_{xx}}{D} W - ia(\beta + \beta^*)\frac{\epsilon_{xz}}{D} H_y H_y^*]\Big|_0^1 \tag{12}$$

and from Eq. 9 we obtain

$$\text{R.H.S. of Eq. 12} = i\omega a \epsilon_o (E_z H_y^* + E_z^* H_y)\Big|_0^1 \tag{13}$$

We note that $\frac{1}{4}$ $(E_z H_y^* + E_z^* H_y)$ is the x-component of the time averaged Poynting vector. Since E_z and H_y are continuous across the surfaces of slab at x=0, 1, and since in the outside isotropic medium there is no propagation of energy in the x direction for frequencies above the cutoff, the R.H.S. of Eq. 12 is equal to zero.

In the L.H.S. of Eq. 12, the expression in the square brackets is proportional to the z component of time averaged Poynting vector. This is not equal to zero, since there must be energy flow in the direction of propagation of the guided wave. Consequently, we must have $\beta = \beta^*$. That is, β must be real.

Eq. 7 describes wave propagation in an anisotropic and inhomogeneous medium; it is a second order differential equation with complex coefficients and an unknown real propagation constant β. We have developed a modified shooting method[7] to solve this equation numerically, where we vary the propagation constant β instead of the slope at the boundary. Using this method, we have solved Eq. 7. We have used the following parameter values:

$\frac{a\sqrt{\epsilon_\perp}}{\lambda} = 1.5$, $\frac{\epsilon_\perp}{\epsilon_c} = 1.03$ and $\frac{\epsilon_\parallel}{\epsilon_\perp} = 1.2844$; λ is the wavelength in free space. The magnitudes of the field distributions for TM_{01} and TM_{02} modes are shown in Fig. 2 and Fig. 4, and the phase distributions for TM_{01} and TM_{02} modes are shown in Fig. 3 and Fig. 5. We have arbitrarily chosen the phase to be zero at $x = 0$. As can be seen from Fig. 4, the phase increases with the position x inside the slab. When $x \geq 1$, the phase no longer changes. For a homogeneous and anisotropic slab, the phase increases linearly with position.[4]

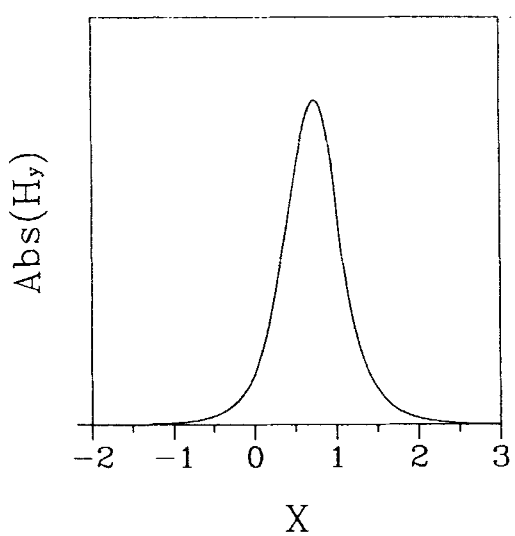

Figure 2. The magnitude of field distribution for TM_{01} mode.

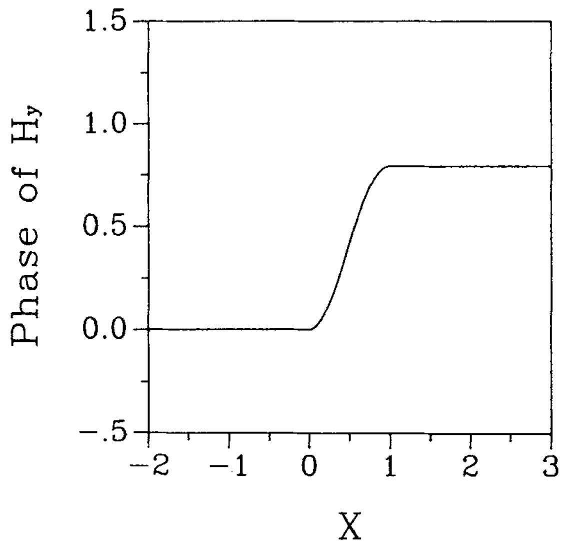

Figure 3. The phase distribution for TM_{01} mode.

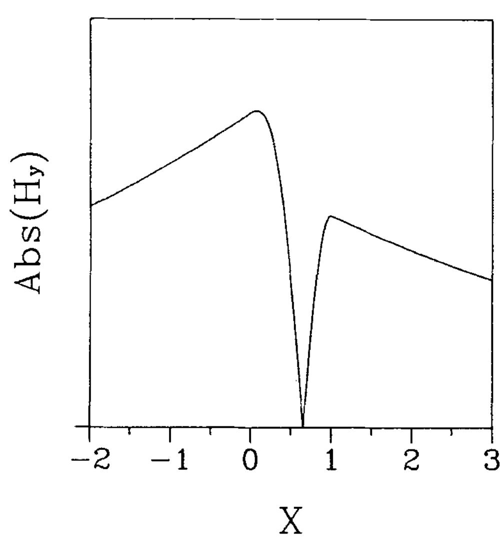

Figure 4. The magnitude of field distribution for TM_{02} mode.

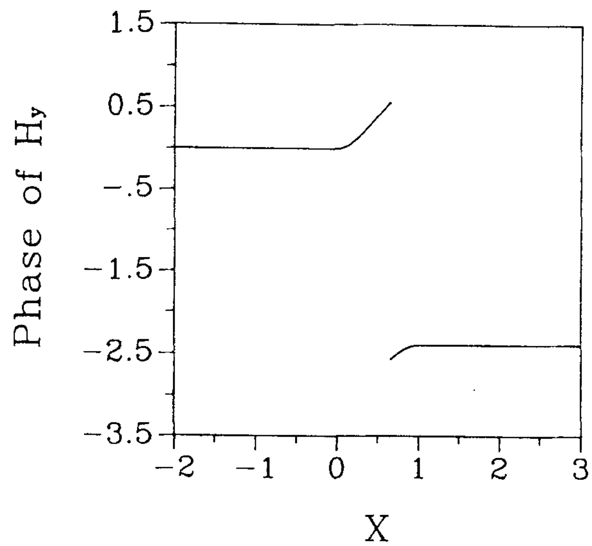

Figure 5. The phase distribution for TM_{02} mode.

We note that for the TM_{02} mode we are very near cutoff; that is, the decay constant outside the slab is close to zero. It is interesting to note that the standard step-index[8] method, where the slab is decomposed into a number of anisotropic homogeneous sub-regions, does not work well in our case, because spatial derivatives occurring in the coefficients of the wave equation are not taken into account. An adaptation of the method to include these terms is straightforward.

CONCLUSION

The anisotropy and inhomogeneity of the liquid crystal layer in a slab waveguide gives rise to complex coefficients in the wave equation. We have shown that a real propagation constant β and corresponding guided modes exist for all frequencies above the cutoff. The resulting complex wave equation has been solved numerically for the TM modes by a novel adaptation of the shooting method. The solutions are interesting in that the phase of H_y varies continuously within the slab.

ACKNOWLEDGMENT

This research was supported in part by the National Science Foundation Science and Technology Center ALCOM DMR89-20147, and by DARPA through US Army CECOM Center for Night Vision and Electro-optics. We acknowledge useful discussions with J. Kelly and H.J. Yuan of the Liquid Crystal Institute.

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